Geometry of Drones: A Unit of Discovery

This unit is designed to review concepts taught throughout the year in Geometry. Students will have already learned the information presented in the curriculum, and they will now have the opportunity to further understand, apply, and experience geometric principles as they are employed through drone technology and design. This plan can be used over 2-4 days (or longer). Pretest and posttest are on page 17 of this document.

STEP 1—Understanding Drone Design—

Oklahoma Academic Standards

- **G.RL.1** Use appropriate tools and logic to evaluate mathematical arguments.
  - **G.RL.1.1** Understand the use of undefined terms, definitions, postulates, and theorems in logical arguments/proofs.

- **G.2D.1** Discover, evaluate and analyze the relationships between lines, angles, and polygons to solve real-world and mathematical problems; express proofs in a form that clearly justifies the reasoning, such as two-column proofs, paragraph proofs, flow charts, or illustrations.
  - **G.2D.1.1** Apply the properties of parallel and perpendicular lines, including properties of angles formed by a transversal, to solve real-world and mathematical problems and determine if two lines are parallel, using algebraic reasoning and proofs.
  - **G.2D.1.2** Apply the properties of angles, including corresponding, exterior, interior, vertical, complementary, and supplementary angles to solve real-world and mathematical problems using algebraic reasoning and proofs.
  - **G.2D.1.3** Apply theorems involving the interior and exterior angle sums of polygons and use them to solve real-world and mathematical problems using algebraic reasoning and proofs.
  - **G.2D.1.4** Apply the properties of special quadrilaterals (square, rectangle, trapezoid, isosceles trapezoid, rhombus, kite, parallelogram) and use them to solve real-world and mathematical problems involving angle measures and segment lengths using algebraic reasoning and proofs.
  - **G.2D.1.5** Use coordinate geometry to represent and analyze line segments and polygons, including determining lengths, midpoints, and slopes of line segments.
  - **G.2D.1.6** Apply the properties of polygons to solve real-world and mathematical problems involving perimeter and area (e.g., triangles, special quadrilaterals, regular polygons up to 12 sides, composite figures).
  - **G.2D.1.7** Apply the properties of congruent or similar polygons to solve real-world and mathematical problems using algebraic and logical reasoning.
  - **G.2D.1.8** Construct logical arguments to prove triangle congruence (SSS, SAS, ASA, AAS and HL) and triangle similarity (AA, SSS, SAS).
  - **G.2D.1.9** Use numeric, graphic and algebraic representations of transformations in two dimensions, such as reflections, translations,
dilations, and rotations about the origin by multiples of 90°, to solve problems involving figures on a coordinate plane and identify types of symmetry

- **G.3D.1** Solve real-world and mathematical problems involving three dimensional figures.
  - **G.3D.1.1** Solve real-world and mathematical problems using the surface area and volume of prisms, cylinders, pyramids, cones, spheres, and composites of these figures. Use nets, measuring devices, or formulas as appropriate
  - **G.3D.1.2** Use ratios derived from similar three-dimensional figures to make conjectures, generalize, and to solve for unknown values such as angles, side lengths, perimeter or circumference of a face, area of a face, and volume.

- **G.C.1** Solve real-world and mathematical problems using properties of circles
  - **G.C.1.1** Apply the properties of circles to solve problems involving circumference and area, approximate values and in terms of \( \pi \), using algebraic and logical reasoning.
  - **G.C.1.2** Apply the properties of circles and relationships among angles; arcs; and distances in a circle among radii, chords, secants and tangents to solve problems using algebraic and logical reasoning.

- **G.RT.1** Develop and verify mathematical relationships of right triangles and trigonometric ratios to solve real-world and mathematical problems.
  - **G.RT.1.1** Apply the distance formula and the Pythagorean Theorem and its converse to solve real-world and mathematical problems, as approximate and exact values, using algebraic and logical reasoning (include Pythagorean Triples).
  - **G.RT.1.2** Verify and apply properties of right triangles, including properties of 45-45-90 and 30-60-90 triangles, to solve problems using algebraic and logical reasoning.
  - **G.RT.1.3** Use the definition of the trigonometric functions to determine the sine, cosine, and tangent ratio of an acute angle in a right triangle. Apply the inverse trigonometric functions to find the measure of an acute angle in right triangles.
  - **G.RT.1.4** Apply the trigonometric functions as ratios (sine, cosine, and tangent) to find side lengths in right triangles in real-world and mathematical problems.

**Materials:**

- Drones—several different sizes, types, brands
- Interactive Notebook
- Paper, construction paper, plain paper, cardstock
• Isometric dot paper
• Isometric graph paper
• Tools for geometric constructions
  o Straightedge
  o Compass
  o Protractor
• Chromebook with a 3D CAD design software (optional)
• Slide presentation
  o https://www.slideshare.net/dwe0008/ultimate-history-of-all-things-drones
• Articles
  o https://www.droneomega.com/types-of-drones/
  o https://thewiredshopper.com/tricopter-versus-quadcopter/
• Graphic (useful)
  o https://makezine.com/2014/01/07/anatomy-of-a-drone/

SWBAT:
• explore the basic design of the drone presented to you
  o What do you notice about the design?
  o How would you describe the drone using academic vocabulary?
  o Any connections to math or scientific principles that you know?
• identify basic geometry terms (vocabulary, undefined terms) displayed and presented in the body and operation of a drone
• Understand objects in 3D
• Understand the role that geometry plays in drone design and technology

Begin with slide presentation https://www.slideshare.net/dwe0008/ultimate-history-of-all-things-drones. Then provide students with drones to examine. Give time to explore.

With your partner, examine the drone you have available. Discuss the 3 questions presented above. Be prepared to give an oral and written description of your drone. Use your notebook to help you recall information presented throughout the year from this class (and other classes you have taken). In what ways is geometry used? Did you find the same elements as your partner or group? Did you identify different aspects?

Remember early in the school year our class practiced representing 3D figures in 2D using nets, isometric dot paper, or graph paper (isometric or plain). Plan and create a 2D representation of your drone, representing your drone as closely as possible.
Geometry curriculum includes many academic vocabulary words that are essential to understand for future math and science classes. Students should show mastery of the geometry vocabulary and identifying geometry in technology and design. A non-exhaustive list of geometry vocabulary words and concepts is provided at the end of the lesson.

In examining your drone, what did the body look like? Are the parts exposed or are all the “inner” components enclosed somehow? How might that affect

- the appeal?
- the performance?

What would we call a drone with 3 rotors? <see article>
https://thewiredshopper.com/tricopter-versus-quadcopter/

https://www.droneomega.com/types-of-drones/
This link is for an article that explains that type is determined by the kind of technology used to keep the drone flying. All are unmanned aerial systems (UAS). Also, the article touches on some pros and cons to designs.

- Fixed wing
- Single rotor helicopter
- Multirotor—named by the number of rotors (not by the number of arms)
  - Tricopters—3 rotors
  - Quadcopters—4 rotors
  - Hexacopters—6 rotors
  - Octocopters—8 rotors
- Fixed Wing Hybrid—being developed as Amazon Prime’s delivery drone

Notice that we call the drones in our class “quadcopters.” Why? <Hopefully a reference is made to the 4 rotors.>

DESIGN CHALLENGE
Focusing on multirotor drones (or either of the other types), students are tasked with designing a drone. Be creative, include changes you would like to have on a drone that is yours. As you design, remember details!! You must include all specifics on measurement of sides, angles, and components. Do not forget

- Every rotor has its own motor and esc (electronic speed controller)
- Rotors need clearance (cannot be too close together)
- Necessary components need to be included in the design
  - https://makezine.com/2014/01/07/anatomy-of-a-drone/
  - Some parts are optional (for example, not all drones have cameras)
- Everyone must have a design/drawing that is unique in some way (no copying, duplicating, etc)
- Done manually (on graph paper) or with CAD
• Describe ways that geometry is used in design
• <Teacher may assign the drone that students design. Include tricopters, quadcopters, hexacopters, octocopters, decacopters, dodecacopters.>
• <Teacher may decide if students are allowed to research options online.>

DESIGN CHALLENGE CONTINUES
Great news—your design caught the attention of a huge company, and they are interested in buying your drone! However, the manufacturer has returned to ask you to make modifications. You must now redesign your drone for a specific purpose. Choose one of the following <teacher can assign—can add to list of possible purposes; teacher can assign as a solo, partner, group project>:
• racing
• surveying storm damage
• transporting medical materials to a remote quarantined area
• crop dusting
• making a movie with severe-angled scenes
• rescuing trapped mountain climbers
• military air strikes
• archeological discovery requiring preservation
• safety inspection of roofs

Assume that you have access to any materials you need, but you may not eliminate essential parts of the drone.

In your redesign, consider
• what changes should be made?
• What considerations should be reviewed?
• How will the new design be different from your original?

Again, you will need to produce a design on paper, graph paper, or a computer file. Care should be taken to ensure that all measurements are close. If possible, consider creating a net or some other 3d representation of your drone.

Presentation of design to class.

<Activity can be expanded to include students designing a way to tether a drone so that it can fly and remain contained and controlled. A photo will be included below to show what was used in implementation of this lesson.>

GPS and drones—circles circles circles—see PDF “GPS and Drones”
TRIALS, TIBULATIONS, TRIANGLES, TRIGONOMETRY, and TETHERING

Additional materials
- Tethering platform
  - Plywood
  - Polystyrene foam insulation board
  - Retractable dog leash
  - Measuring tape
  - Calculator with trigonometric functions

Students will need to measure
- How tall they are
- How long is the leash (a 10-foot length is used in example)
- How high is the tethering platform
For this lesson, a drone can be used to make the activities much more interesting and engaging for students. Groups of 3-4. Each student does their own calculations. Students should be able to explain all calculations and procedures.

1. Set hovering height of the drone using Ardupilot or appropriate app (drone set at A). (AD) ***Each student should do a variation of the measurements—work together to collaborate on the calculations and methods, but using their own unique measurements within the group. Example: Student 1 sets hover height at 12 feet and stands 12 feet from the launch point. Student 2 could have a multiplier of ½, so he will set hover at 6 feet and stand 6 feet from the launch point. Student 3 could have a multiplier of ¼, so her hover height will be 3 feet, and she will stand 3 feet from the launch point. ***All of student 1’s measurements should be multiplied by the multiplier, including height of eye level.

2. How far are you standing from the launch pad? (FD or BC)

3. What is the distance from the ground to student eye level? (BF)

4. What is the angle at C if the launch is straight up or vertical? <right angle or 90>

5. Name the rule that says, “In a plane, if a line is perpendicular to one of two parallel lines, then it is perpendicular to the other line also.” <Perpendicular Transversal Theorem>

6. So, if BC \(// FD\) and \(C=90\), then \(D=90\).

Launch drone to set height. Hover as you answer the following:

7. How far is the drone from your eyes? (AB)

8. Using graph paper, represent the scenario. <students should draw right triangles appropriately.> \(ABC\)

9. What is the scale of your representation in #8 above? <for example, 1 unit on graph paper = 1 foot>

10. What is the distance from drone to your foot? (AF)

11. Determine the angle of elevation from eye to drone. (use \(\tan B\) to find \(B\))

12. Determine the angle of depression from drone to eye. (\(B\) of depression... ..because those are Alternate Interior Angles...)
13. Determine the angle of elevation from foot (ground) to drone. (*F*)

14. Determine the angle of depression from drone to foot.
15. How will you find \( DAF \) and \( CAB \)? Explain how, then calculate those angles.

16. What figure did you create in your representations of this exercise? Is it a 2D or 3D?  
<triangle, 2D>

17. Name the plane(s) where your figure is found. <Plane \( ABC \); any 3 letters> How many planes are there that include your figure? <one plane, can be named various ways>

18. What is the area of the figure created by the drone, the launch point, and the student shoe? <\( ADF \) (life sized)>

19. What is the area of the figure drawn on the graph paper? <\( ADF \), but on paper will be much smaller>

20. Is the graphed area congruent to the figure created by drone, launch point, and shoe? <no>

21. Is the graphed figure similar (proportional)? <should be> What is the scale factor?  
<area \text{ paper}/area \text{ actual}>

22. With the adjustment of the hovering height and standing distances,
a. What happens to the measurements of the triangle sides? <those change proportionally>

b. What happens to the measurements of the angles? <angles stay the same measurement in similar triangles>

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The following questions apply to measurements taken when flying the drone as it is tethered to a short platform designed by Todd Dixon and modified by Leslie Thornburgh. A protractor will be fitted to the platform so that the angle of the tether can be determined.

1. Determine length of tether (retractable leash). (Mine is 10 feet)

2. Measure distance from tether to student.
3. Launch drone and allow tether to be extended fully.

4. Read protractor to find $C$.

5. Students should now have 2 sides and an angle. Use Law of Cosines to find missing side $c$.

6. Students should now have 3 sides and an angle. Use Law of Sines to set up proportion and solve for missing angles.

**Law of sines**

\[
\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}
\]

**Law of Cosines**

\[
\begin{align*}
a^2 &= b^2 + c^2 - 2bc\cos A \\
b^2 &= a^2 + c^2 - 2ac\cos B \\
c^2 &= a^2 + b^2 - 2ab\cos C
\end{align*}
\]
1. Launch drone until tether is fully extended. It should hover at an acute angle (not completely vertical), and the angle should be determined by measuring with the protractor. Student will now know the measurement of the tether and the acute angle referenced in the graphic.

2. Using controller, fly drone 360. What figures are created? (Note that these will require a little bit of imagination... Circle? Cylinder? Cone? Use trig functions to find the radius. (From tether at the base to the imaginary vertical.)

3. Are the figures being represented 2D or 3D figures?
4. Find surface area and volume of all figures created.

5. Is the student able to “see” the cone created? Is the student able to “see” the cylinder created? Students should sketch the figures. Is the student able to visualize a cylinder that is missing the inner cone? Find the surface area and volume of the yellow portion in the graphic below... visualize the yellow filling the cylinder to the edges—the grey is the cone that has been removed from the center.

Using the same tethered drone, fly the drone directly vertical, and hover at that location for the remainder of these math problems. What shape is created??
Connect (imagination again) the drone to each of the 4 corners...what shape is created?
Pyramid. Remember all of the steps and calculate surface area and volume.

<Extend activity—have students take a photo with the drone camera, then use that photo in an activity involving proportions or area.>

<Extend activity—change the height of the hovering drone, which will create a shorter pyramid. How are the surface area and volume changed? Is the change proportional? Are similar figures created?>
As an added facet, students could be required to represent these 3D figures on Solidworks or other CAD program...or a hand-drawn representation.

Conclusion—
Drones are becoming more visible and useful in our society, and they can even be used to make learning math FUN! Adding drones to the ongoing curriculum will add engagement and interest, which should serve to enhance the learning process.
Pre Test/Post Test

1. In what ways does geometry show up in your everyday life?

2. In what ways might a drone be useful in studying geometry?

3. How might drones vary in design and purpose?

4. How might geometry relate to GPS?

5. What are the best ways to represent 2D and 3D figures?
VOCABULARY

Angle bisector
Congruent segments
Construction
Isometric drawing
Linear pair angles
Net
Orthographic drawing
Perpendicular bisector
Postulate
Segment bisector
Supplementary angles
Vertical angles
Point
Line
Plane
Collinear points
Coplanar
Space
Segment
Ray
Opposite rays
Postulate
Axiom
Intersection
Coordinate
Distance
Congruent segments
Midpoint
Angle
Sides of an angle
Vertex of an angle
Measure of an angle
Acute angle
Right angle
Obtuse angle
Straight angle
Congruent angles
Adjacent angles
Complementary angles
Linear pair
Straightedge
Compass

Construction
Perpendicular lines
Perimeter
Circumference
Area
Inductive reasoning
Conjecture
Counterexample
Conditional
Hypothesis
Conclusion
Truth value
Negation
Inverse
Contrapositive
Equivalent statements
Biconditional
Deductive reasoning
Law of detachment
Law of Syllogism
Reflexive Property
Symmetric Property
Transitive Property
Proof
Two-column proof
Theorem
Postulate
Parallel lines
Skew lines
Parallel planes
Transversal
Alternate interior angles
Same-side interior angles
Corresponding angles
Alternate exterior angles
Auxiliary line
Exterior angle of a polygon
Remote interior angles
Slope
Slope-intercept form
Point slope form
Congruent polygons
Polygon

Scalene
Isosceles
Equilateral
Equiangular
Legs of an Isosceles
Congruent Triangles
Base of an Isosceles
Vertex angle
Base angles
Corollary
Side Side Side
Side Angle Side
Angle Side Angle
Angle Angle Side
Hypotenuse Leg
Midsegment
Equidistant
Point of concurrency
Circumscribed
Circumcenter
Incenter
Inscribed
Median
Centroid
Altitude
Orthocenter
Polygon
Equilateral polygon
Regular polygon
Parallelogram
Consecutive angles
Rhombus
Square
Rectangle
Kite
Trapezoid
Radical
Similarity
Proportional
Ratio
Indirect Measure
Extended proportion
Scale factor
Scale drawing
Scale
Angle Angle
Side Side Side
Side Angle Side
Fractal
Geometric Mean
Golden Ratio
Pythagorean Theorem
Pythagorean Triple
45-45-90
30-60-90
Trigonometry
Trigonometric ratios
Sine
Cosine
Tangent
Angles of Elevation
Angles of Depression
Law of Sines
Law of Cosines
Vectors
Transformation
Preimage
Image
Isometry
Translation
Compositions
Reflection
Line of Reflection
Symmetry
Line Symmetry
Reflectional Symmetry
Line of Symmetry
Rotational Symmetry
Point Symmetry
Rotation
Center of Rotation
Angle of Rotation
Dilation
Center of Dilation
Enlargement
Reduction
Glide Reflection
Tessellation
Height
Area
Perimeter
Surface area
Lateral area
Base (Capital B)
Apothem
Radius
Circle
Center
Diameter
Congruent circles
Central angle
Semicircle
Minor arc
Major arc
Adjacent arcs
Circumference
Pi
Concentric circles
Arc length
Congruent arcs
Sector of a circle
Geometric Probability
Polyhedron
Face
Edge
Cross section
Prism
Cylinder
Right prism
Oblique prism
Right cylinder
Oblique cylinder
Pyramid
Regular pyramid
Cones
Right cone
Volume
Composite
Sphere
Center of a sphere
Center of a sphere
Diameter of a sphere
Great circle
Hemisphere
Tangent to a circle
Point of tangency
Chord
Arc
Inscribed angle
Intercepted arc
Secant
Standard form
Equation of a circle
Locus